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Measurement Uncertainty in Testing

**A short introduction on how to characterise
accuracy and reliability of results
including a list of useful references**

Technical Report

This short introduction to measurement uncertainty and its implementation into the laboratory as requirement for accreditation according to ISO/IEC 17025 is intended to provide help for the inexperienced rather than the expert and therefore necessarily simplifies some topics.

The document is in the progress state and it is intended, to add further examples from non-chemical fields of testing. Proposals for such examples are very welcome.

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1. Some aspects of measurement uncertainty

What is measurement uncertainty ?

- Measurement results are never exact, nor absolutely free of doubts. Therefore the “measurement uncertainty” is part of the result of a measurement. It is a measure for the accuracy of the result.
- Measurement uncertainty is derived from standard deviations.
- Definition: Measurement uncertainty is “A parameter associated with the result of a measurement, that characterises the dispersion of the values that could reasonably be attributed to the measurand” (VIM¹ and GUM [1])

Who needs measurement uncertainties ?

- The customer needs to get an idea of the “accuracy” of the result,
- measurement uncertainty has to be taken into account particularly when regarding specification limits² (⇒ legal and liability aspects),
- testing laboratories need uncertainties with their calibration certificates, so that they can state the uncertainty of their own measurements.

Where do measurement uncertainties in testing come from ?

- There are many possible sources of uncertainty, e.g. sampling, instrument drifts and calibration, homogenisation and dilution effects, human factors, environmental effects, ...

Who needs to give measurement uncertainties ?

- An estimation of a measurement’s uncertainty is required for testing and calibration laboratories complying with ISO/IEC 17025.
5.4.6.1: “A calibration laboratory ... shall have and shall apply a procedure to estimate the uncertainty of measurement for all calibrations ...”,
5.4.6.2: “Testing laboratories shall have and shall apply procedures for estimating uncertainty of measurement”
- Whether those uncertainties have to be stated in the test report depends on requirements by the test method, requirements by the customer, or whether conformance to specification has to be assessed (ISO/IEC 17025, 5.10.3).
- In calibration, uncertainties have to be stated in the certificate (as they are required by the user of the calibrated equipment).

How to obtain and state measurement uncertainties

- There are clear guidelines for
 - calculating / estimating uncertainties for each source separately, then
 - combining of the contributions from each uncertainty source and finally
 - stating the uncertainty of a result.
- These guidelines are given in the GUM, “Guide to the expression of uncertainty in measurement” [1], the “main book” of measurement uncertainty, edited by ISO, harmonising internationally the estimation and reporting of measurement uncertainties.
- The basis of any evaluation of measurement uncertainty is a statistical approach.
- However, it may be implemented in facilitated ways, e.g. by estimating the “overall uncertainty” involving precision and/or validation data³ available in the laboratory.

¹ Vocabulary of Basic and General Terms in Metrology, ISO, Geneva, 1993, ISBN 92-67-10175-1

² e.g. ILAC G8 [69]

³ e.g. ISO/IEC 17025 (5.4.6) citing ISO 5725 [29]

How are measurement uncertainties expressed ?

- Report whether a single standard deviation is used or whether an expanded uncertainty with the respective coverage factor and level of confidence is stated with the result.
- Example: "Height 20.051 ± 0.022 cm. The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor $k = 2$, providing a level of confidence of approximately 95%."
- It may be useful to (briefly) state how the uncertainty was obtained and what it includes.
- Two significant digits [1] (unless there are other requirements).
- The statement must never give a false positive impression of the uncertainty associated with the measurement (ISO/IEC 17025, clauses 5.4.6.2 and 5.4.6.3).

2. Determination of measurement uncertainty - ways for estimating uncertainties in practice

First of all: a laboratory that has a good quality management system should have little effort to state the uncertainty of a result.

The principles for correct application of measurement uncertainties are given in the GUM [1]. For further reading:

- The guide for "Quantifying Uncertainty in Analytical Measurement" by EURACHEM /CITAC [3] can be highly recommended for (analytical) chemists.
- Good explanations and examples from the calibration field are also found in "Guidelines to the Expression of the Uncertainty of Measurements in Calibrations" [2].

Besides these technical papers ILAC (International Laboratory Accreditation Co-operation) published the strategy paper ILAC-G17:2002 "Introducing the Concept of Uncertainty of Measurement in Testing in Association with the Application of the Standard ISO/IEC 17025" [20] which is also applied by EA (European Co-operation for Accreditation). Asia Pacific Laboratory Accreditation Cooperation (APLAC), too, has published a draft policy [21], taking into account sector-specific requirements.

Chapter 3 gives a short summary of the procedure for estimation of the test result and the accompanying measurement uncertainty in 8 steps as described in the GUM (chapter 8) [1]. GUM groups uncertainty components into type A and type B according to the way these data were obtained. Type A components are calculated by statistical means from repeated measurements while type B components are taken from other sources e.g. manufacturer's manuals, validation information or average control charts. For further details see chapter 3.

Besides this mathematical analytical approach also more pragmatic approaches are in conformity with GUM (and also in conformity with the requirements of ISO/IEC 17025). Therefore before starting the procedure of uncertainty determination it is worth looking for all information available, which might reduce the effort for the uncertainty evaluation. The aim is to find a way fit for purpose.

Such information can be grouped data, combining the contribution from several uncertainty components, like the standard deviation within interlaboratory comparisons. From this data it may be possible to already estimate the "overall uncertainty". Also using uncertainty data that have been assessed by type B estimation may simplify the approach. As stated in GUM, type A and type B uncertainty components are of the same nature and value. For example type B might be even better than type A when only a few repetitions have been performed.

However, before using these data it has to be checked whether the conditions apply.

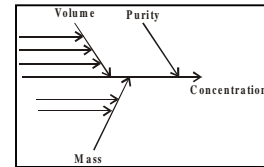
With this information in hand the operator will be able to decide, which of the steps of uncertainty estimation are still necessary for the specific experiment.

Estimating uncertainties in practice

There are different possibilities to estimate measurement uncertainty budgets. The strict mathematical way is described most extensively in the GUM, but the other methods are, too, well compliant with the GUM.

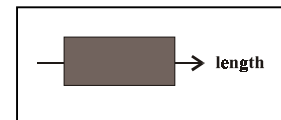
In the strict mathematical analytical method (see chapter 3),

- all components are estimated separately and
- the single uncertainties are combined applying the law of propagation of uncertainty.



Another possibility is to already use grouped data, in order to directly estimate the “overall-uncertainty”,

- e.g. from quality assurance data that are available in the laboratory (e.g. standard deviations from repetition of all steps (this would be type A) or precision data⁴, e.g. the standard deviation of an interlaboratory comparison (type B))
- where all uncertainty components are included, but not evaluated separately. In this respect this is a „black-box“-approach.



However, in practise mostly a **combined approach** will be the most suitable, in which some components or steps are grouped. This combined approach will apply very often, as it is often impossible to estimate each uncertainty individually.

The data available in a laboratory will often be data from quality control which are commonly in the form of precision data⁴, i.e. reproducibility or repeatability standard deviations.

The use of precision data for uncertainty estimation is described in the French standard FD X 07-021 [15], the Eurachem Guide [3] and a draft paper prepared by ISO TC69/SC6/WG7 [34]. Precision data are described in ISO 5725 [29].

It is important to stress, that using practically obtained data for an “overall uncertainty”, like standard deviations within control charts, from interlaboratory comparisons or validation data, may be easier. However, this approach is not a priori less valid. Experience shows that uncertainty estimations obtained by the mathematical analytical approach are often too small. This is due to the fact that it is difficult to draw a comprehensive model equation and therefore there is a tendency to omit some significant components. Therefore any measurement uncertainty estimated by the mathematical approach should always be compared with practical data. Ideally of course, measurement uncertainties obtained either way should be of the same size.

How to use existing quality assurance data

▪ Using data stated in standards

Some written standards specify the form of presentation or define how to state the measurement uncertainty. When the laboratory demonstrates its ability to use the standard method properly, it may give the result as stated in the standard.

In other cases the method validation data are given in the standard, e.g. through interlaboratory comparison. These data may be also used for estimation of the measurement uncertainty.

EUROLAB demands to include such uncertainty statements in all new standards.

⁴ Precision data may be obtained under repeatability or reproducibility conditions (compare ISO 5725)

- **In-house methods**

In-house methods have to be validated by the laboratory (ISO/IEC 17025, 5.4.5). Many validation methods imply measurements from which the standard deviation may be obtained and used for uncertainty estimation of the method. Reference materials and quality control materials may be of great help in such a validation process.

- **Interlaboratory comparisons**

Interlaboratory comparisons are a useful tool in uncertainty evaluation. The series of measurements of the different laboratories delivers the results from a great number of independent measurements under different conditions and provides the standard deviations of them. These data may well be used by a laboratory (having performed satisfactorily) as the standard uncertainty of the analysed parameter, provided that the comparison covers all relevant uncertainty components and steps (ISO/IEC 17025, 5.4.6.3). It should be stressed that for the uncertainty estimation the standard deviation from all participating laboratories should be used and not the difference of the single laboratory's result from this standard deviation.

- **Average control charts**

Average control charts are used by laboratories as a quality tool for instruments and methods. The standard deviation s is determined in the charts (e.g. to set the 2s warning- and 3s control limits) and can be used directly as input for uncertainty evaluation (see Example 1b). However, one should consider whether relevant reproducibility elements of uncertainty may be missing.

- **Estimation by experts**

Estimation by experts on a certain experiment often reflects the uncertainty of an experiment very well, resulting from experience and knowledge. It is very difficult though to give advice on this source of uncertainty determination. The bottom line is that the laboratory should be able to demonstrate that it performs within that uncertainty estimate.

No actions for uncertainty determination are required:

- at this time for qualitative and semi-quantitative methods, or
- where a well-recognised test method specifies the limits of the values of the major sources of uncertainty of measurement and specifies the form of presentation of calculated results. In this case the laboratory is considered to have satisfied uncertainty of measurement requirements by following the test method and the reporting instructions [21].

The laboratory should ...

- check
 - whether all relevant uncertainty sources are considered: e.g. sampling, preparation, dissolving, dilution
 - whether the conditions of the quality assurance data are comparable with the treated problem, e.g. matrix and composition, range of values, repeatability / reproducibility contributions
 - whether the presumed simplifications are tolerable
 - whether it is a field of testing with special requirements (reference materials, health, safety, ...)
- give a clear indication which steps are not included in the uncertainty budget
- start with the data available in the laboratory and strive for continuously improving the uncertainty statements with increasing number of data available and state of knowledge.

Aim: fit for purpose

The most important rule is: effort and expenditure for determination of uncertainties should be clearly guided by the principle "fit for purpose", that is, it should be good enough to meet the requirements of the user of the measurement data, but do not overdo it!

For testing laboratories the calculation of uncertainties following the strict mathematical approach involving all 8 steps (as described in chapter 3), carrying out several measurements for each uncertainty source and involving the complicated mathematical equations for calculation of the uncertainty, may not be efficient or even applicable.

Instead, the experimenter should try to identify the relevant sources of measurement uncertainty. This information will help him to improve the quality system. Therefore the experimenter should try to find sources of information, which are of key importance for controlling the measurement procedure (system), thus allowing to implement the evaluation procedure most efficiently.

Using grouped data or "overall uncertainties" are possible ways to estimate measurement uncertainties on a practical basis. Furthermore the GUM [1] encourages Type B evaluation of uncertainties from existing data, which usually reduces the expenses considerably, recognising that Type B evaluation can be as reliable as Type A evaluation (4.3.2).

3. The mathematical analytical way: 8 steps to obtain test result and uncertainty

This procedure for evaluation and expressing uncertainty is described in detail in the GUM [1]. Before start: Check for all possible information simplifying the uncertainty determination

- Step 1) Specify measurand, express mathematically the equation relating measurand and input quantities. Identify all uncertainty sources.
- Step 2) Determine the input quantities.
- Step 3) Quantify the standard uncertainties of all single components.
- Step 4) Identify the covariances (of correlated input quantities).
- Step 5) Calculate the result of the measurement from the input quantities.
- Step 6) Calculate the combined uncertainty.
- Step 7) Calculate the expanded uncertainty.
- Step 8) Give the result together with the uncertainty as estimated.

The single steps are carried out as follows:

Step 1) Specify measurand

Express mathematically the equation relating measurand, i.e. the quantity subject to measurement, and input quantities. Identify all uncertainty sources.
A good way for listing the components is in "cause and effect diagrams" [3].

The equation will be of the form of a function $f: y = f(x_1, x_2, x_3, \dots, x_N)$, expressing that the measurand (result) is defined by a function depending on the N single quantities $x_1, x_2, x_3, \dots, x_N$.

Step 2) Determine the input quantities $x_1, x_2, x_3, \dots, x_N$. Determine the values of the input quantities, e.g. through measurement, from tables and from specifications.

Step 3) Quantify the standard uncertainties of all single components

There are two ways to determine the single uncertainties:

- a. Calculating it by statistical means from repeated measurements (Type A)
- b. Estimating values from other sources (called Type B)

Type A: Standard uncertainties calculated from a series of n measurements.

The following mathematical formulas have to be applied.

$$x_j = \bar{q} = \frac{1}{n} \sum_{i=1}^n q_i = \frac{q_1 + q_2 + q_3 + \dots + q_n}{n}$$

arithmetic mean (or average) of a series of n independent measurements with
 n : number of independent measurements
 q_i : value of a measurement for determination of the input quantity x_j

$$s(q_i) = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^n (q_i - \bar{q})^2}$$

experimental standard deviation of a single measurement

$$s(\bar{q}) = \frac{s(q_i)}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n (q_i - \bar{q})^2}$$

experimental standard deviation of the mean

$u(x_j) = s(\bar{q})$
 the standard deviation will be used directly as standard uncertainty of the respective single component x_j , measured n times

Type B evaluation of standard uncertainties:

This method uses prior information like: reported uncertainties of a reference material, calibration certificates, previous measurement data, accepted values of constants, experience on behaviour of similar materials and equipment, resolution, instability, environmental conditions, results from interlaboratory comparisons, average control charts. Note: prior information may have been originally derived by statistical methods.

In principle we are always looking for standard deviations (or their squares, called variances).

If values for uncertainties for single components are given as standard deviations of a measurement they can be directly used as $u(x_j)$. For example a standard deviation in an interlaboratory comparison is reported as the single standard deviation s , which can be directly used as standard uncertainty $u(x_j) = s$.

In other cases the statistical distributions have to be taken into account. Often it will then be a bit more difficult when there are statements like "at a level of confidence of 95%⁵" or "20.051 ± 0.022 cm" with ± 0.022 being the maximum deviation expected. In the first case normal distribution is presumed, in the second case the operator has to decide whether rectangular or triangular distribution or others apply:

⁵ the required level of confidence will vary for different fields: while 95% may be sufficient in a technical field of testing, 99.7% may be required e.g. in the health sector and for legal applications.

<p>Normal distribution / confidence intervals given: e.g. if a calibration certificate states the confidence interval to be 95%, the uncertainty of that component is derived through:</p>	$u(x_j) = \frac{\text{expanded uncertainty}}{k}$ <p>(with $k=2$ ca. 95% confidence, $k=3$ ca. 99.7% confidence).⁵</p>
<p>Rectangular distribution: Upper and lower limits (a_- and a_+) are given: (example: last digit of a digital display)</p>	$u(x_j) = \frac{(a_+ - a_-)/2}{\sqrt{3}}$ <p>if $a_+ - a_- = 2a_j$ then $u(x_j) = \frac{a_j}{\sqrt{3}}$</p>
<p>Triangular distribution: applies, if values close to the measured value (centre) are more likely than the values close to the limits. (Example: volume of a flask: 100 ml \pm 0.1 ml; volumes close to the nominal value are more likely than the extremes)</p>	$u(x_j) = \frac{(a_+ - a_-)/2}{\sqrt{6}}$ <p>if $a_+ - a_- = 2a_j$ then $u(x_j) = \frac{a_j}{\sqrt{6}}$</p>

$u(x_j)$
the standard uncertainty is derived from prior information either directly or after consideration of the statistical distribution

Step 4) Identify the covariances (of correlated input quantities)

If two input quantities have a common source of uncertainty (e.g. they both depend on an uncertain temperature), their uncertainties are not independent and therefore they are said to be *correlated*.

For further details see paragraph F.1.2.3 of the GUM [1].

In practice, correlation effects often are not known and the approximation (or assumption) is made that there is no correlation.

Note: Not taking into account existing correlations may result in a wrong estimate of measurement uncertainty.

Step 5) Calculate the result of the measurement from the input quantities

The result is: $y = f(x_1, x_2, x_3, \dots, x_N)$

Step 6) Calculate the combined uncertainty from the uncertainties of the single components (as determined in 3), taking into account possible covariances.

The combined uncertainty is the square root of:

(first order Taylor approximation) for non-correlated uncertainty components	$u_{combined}^2(y) = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$
[For reasons of completeness the equation for correlated uncertainty components is also given:]	$u_{combined}^2(y) = \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial f}{\partial x_i} \right) \left(\frac{\partial f}{\partial x_j} \right) u(x_i, x_j)$

In those equations the partial derivative $\left(\frac{\partial f}{\partial x_i} \right)$ is called sensitivity coefficient.

The equations described above look very complicated to most experimenters. In practice they become much simpler for some simple mathematical relations (in case that no correlations exist):

for equations of the measurand involving only sums or differences: $y = x_1 + x_2 + x_3 - x_4 \dots$	$u_{combined}(y) = \sqrt{u_1^2 + u_2^2 + \dots + u_n^2}$
for equations involving only products or quotients: $y = x_1 \cdot x_2 \cdot x_3 / x_4 \dots$	$\frac{u_{combined}(y)}{ y } = \sqrt{\frac{u_1^2}{x_1^2} + \frac{u_2^2}{x_2^2} + \dots + \frac{u_n^2}{x_n^2}}$
for equations of the measurand involving exponents: $y = x_1^a x_2^b \dots x_n^z$	$\frac{u_{combined}(y)}{ y } = \sqrt{\frac{a^2 u_1^2(x_1)}{x_1^2} + \frac{b^2 u_2^2(x_2)}{x_2^2} + \dots + \frac{z^2 u_n^2(x_n)}{x_n^2}}$

A useful help for calculation of difficult uncertainty problems is the spreadsheet method as described in [3] using numerical differentiation.

Step 7) Calculate the expanded uncertainty

The expanded uncertainty, denoted by U , is a measure of uncertainty, given for the appropriate distribution function of the result (e.g. normal distribution), for a given level of confidence (e.g. 95%). The result of a measurement is then conveniently expressed as $Y = y \pm U$. This is interpreted that the best estimate of the value attributable to the measurand Y is y and the interval $y-U$ to $y+U$ may be expected to encompass a large fraction p of the distribution of values that could reasonably be attributed to Y . p is the *probability* or *level of confidence* of the interval.

$$Y = y \pm U = y \pm k \cdot u_{combined}(y)$$

$$U = k \cdot u_{combined}(y)$$

U : expanded uncertainty

k : coverage factor

$k = 2 \Rightarrow$ level of confidence of 95%^{6,5}

$k = 3 \Rightarrow$ level of confidence of 99.7%

Alternatively, **Student distributions**⁷, involving degrees of freedom are sometimes applied, especially when the number of replicates of the measurement carried out is small.

Step 8) Give the result together with the uncertainty as estimated

Results should be given in the form of the following statement:

"Gas pressure: 200.3 ± 2.2 Pa. The reported *expanded uncertainty* is based on a standard uncertainty multiplied by a *coverage factor* $k = 2$, providing a *level of confidence of approximately 95%*."

Two significant digits should be given [1] (unless more digits are required to avoid a loss of information). It may be useful to state briefly how the uncertainty was obtained. However, if any steps or relevant components (sampling, preparation steps, actual measurement,...) are not included, this should be indicated.

⁶ if normal distribution may be assumed

⁷ Student- or t-distribution, see also GUM [1] G.3

4. Examples for estimating measurement uncertainty

The following examples show possibilities for estimation of uncertainties in different testing fields.

Example 1 shows sources of data a laboratory could look for trying to estimate the uncertainty of the measurement of 100 mg sulfate in waste water determined with ion-chromatography. The data available in the laboratory may be the method standard, proficiency test results or control charts.

It is the task of the laboratory to decide, which data are best suitable and which data may not be applicable for reasons of matrix differences, different range of values or because not all uncertainty components may be covered.

Example 1 Chemical Testing

1.a) Estimation of the uncertainty of the measurement of 100 mg sulfate in waste water determined with ion-chromatography from proficiency test results.

(Step 3) The laboratory has participated satisfactorily in the 4th all-German waste-water proficiency testing. The standard deviation of all of the laboratories' results was **s = 4%**.

(Step 6) $s = u = 4\% \Rightarrow u = 4.0 \text{ mg/l}$ for 100 mg/l sulfate

(Step 7) $U_{\text{expanded}} = k \cdot u = 2 \cdot 4.0 \text{ mg/l} = 8.0 \text{ mg/l}$

(Step 8) Sulfate: 100.0 mg/l or: Sulfate: 100.0 ± 8.0 mg/l
measurement uncertainty: 8.0 mg/l (k=2)

This measurement uncertainty was derived from interlaboratory comparison results. It is expressed as an expanded uncertainty and was obtained by multiplication of the combined uncertainty with the factor $k = 2$, relating to a level of confidence of 95%.

Note: Some publications e.g. [34] promote the approach of combining repeatability and reproducibility uncertainties: $u_{\text{combined}} = \sqrt{u_{\text{repeat}}^2 + u_{\text{reproduce}}^2}$, which is too pessimistic. The above method is to be preferred, and sufficient if the proficiency test covers all relevant uncertainty components.

1.b) Estimation of the uncertainty of the measurement of 100 mg sulfate in waste water determined with ion-chromatography from control-charts.

(Step 3) The laboratory maintains an individual control chart for determination of sulfate. The standard deviation from it is **s_{CC} = 3.8%**.

While the control sample of 5 mg/l is measured directly, the sample of 100 mg/l had to be diluted. This dilution step has to be taken into account for the uncertainty budget. The dilution step with a dilutor was carried out and weighed 10 times and delivered a standard deviation **s_{Dil} = 1%**.

(Step 6) $u_{\text{combined}} = \sqrt{u_{\text{CC}}^2 + u_{\text{Dil}}^2} = \sqrt{3.8^2 + 1^2} = 3.9\%$

$u_{\text{combined}} = 3.9 \text{ mg/l}$ for 100 mg/l sulfate

(Step 7) $U_{\text{expanded}} = k \cdot u_{\text{combined}} = 2 \cdot 3.9 \text{ mg/l} = 7.8 \text{ mg/l}$

(Step 8) Sulfate: 100.0 mg/l
measurement uncertainty: 7.8 mg/l (k=2)

This measurement uncertainty was obtained from the laboratory's control charts and is expressed as an expanded uncertainty ...

Note:

This is an example where an additional step has to be considered in addition to quality assurance data that were available in the laboratory. No allowance has been made for any laboratory bias (which is normally required).

1c) Estimation of the uncertainty of the measurement of 100 mg sulfate in waste water determined with ion-chromatography from validation data in the method standard.

(Step 3) The validation data for determination of sulfate in wastewater with ionchromatography are given in ISO 10304-2. In an interlaboratory comparison of an industrial waste water for method validation the reproducibility standard deviation was **6.1%**.

(Step 6) $s = u = u_{\text{combined}} = 6.1 \% \Rightarrow u_{\text{combined}} = 6.1 \text{ mg/l}$ for 100 mg/l sulfate

(Step 7) $U_{\text{expanded}} = k \cdot u_{\text{combined}} = 2 \cdot 6.1 \text{ mg/l} = 12.2 \text{ mg/l}$

(Step 8) Sulfate: 100 mg/l

measurement uncertainty: 12 mg/l (k=2)

This measurement uncertainty was derived from method validation data in accordance to ISO 10304-2. ...

Note: The laboratory must have proved to be able to perform in accordance with this standard method.

Example 2 Mechanical Testing

Hardness testing according to Brinell

At the moment it is difficult to calculate the measurement uncertainty in the field of mechanical testing. This is because of the lack of knowledge how to use the approaches developed in various other fields of chemistry or calibration. Nevertheless there are possible way of estimating the measurement uncertainty to comply with the demands of clients and test standards. The first area in mechanical testing for which a complete system was evaluated is the following example which soon should be a part of the test standards.

This examples can be used to calculate the measurement uncertainty in hardness testing.

This is the first approach how to do this calculation. It is expected that this model will be used in the ISO standards 6506, 6507 and 6508 in the near future. This model was verified using the data of about 95 laboratories, participating in an European proficiency test.

The measurement uncertainty calculated is based on, hardness testing according to Brinell. For this test it is measured how a ball can deform a sample plate.

This example was calculated on the basis of a hardness reference block (CRM) 246.8 ± 1.5 HBW 2.5/187.5 according to the test standard EN ISO 6506. The block was certified by the MPA NRW Dortmund.

The single values of the calibration were:

246.9	245.8	246.3	247.9	247.0
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HBW 2.5/187.5, measured by MPA NRW,

resulting in a standard deviation of $s_{X_{CRM}} = 0.77$ HBW 2.5/187.5.

The uncertainty was estimated according to:

$$U_{\text{expanded}} = 2 * \sqrt{u_E^2 + u_{X_{CRM}}^2 + u_{CRM}^2 + u_H^2 + u_x^2}$$

The single uncertainty components are listed below.
The evaluated uncertainty of 2.1% seems to be reasonable.

STEP	Sources of uncertainty	Abbreviation	Formula	Literature/Certificate	Sample calculation [.] = HBW 2.5/187.5
1	Uncertainty according to the maximum permissible error	u_E	$u_E = \frac{u_{E,2r}}{2.8} (S_R = 2.8 \cdot S_r)$	$u_{E,2r}$ acc. EN ISO 6506-2, Table 2: 2% for this diameter of the ball	$u_E = \frac{0.02 \cdot 246.8}{2.8} = 1.763$
2	Uncertainty of the standardizing machine for calibration of CRM	$u_{\bar{X}_{CRM}}$	$u_{\bar{X}_{CRM}} = \frac{u_{\bar{X}_{CRM,2\sigma}}}{2}$	$u_{\bar{X}_{CRM,2\sigma}}$ acc. calibration certificate of CRM	$u_{\bar{X}_{CRM}} = \frac{1.5}{2} = 0.75$
3	Mean value and standard deviation of the calibration of CRM	\bar{X}_{CRM} , $s_{X_{CRM}}$	$\bar{X}_{CRM} = \frac{1}{n} \cdot \sum_{i=1}^n X_{i_{CRM}}$ $s_{X_{CRM}} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_{i(CRM)} - \bar{X}_{CRM})^2}$	X_i acc. calibration certificate of CRM	$\bar{X}_{CRM} = 246.8$ $s_{X_{CRM}} = 0.77$
4	Standard uncertainty of CRM	u_{CRM}	$u_{CRM} = \frac{t^* s_{X_{CRM}}}{\sqrt{n}}$	$t=1.15$ (Student-factor) for $n=5$ and 68.3% confidence level	$u_{CRM} = \frac{1.15 \cdot 0.77}{\sqrt{5}} = 0.39$
5	Mean value and standard deviation of the measurement on CRM	\bar{H} , s_H	$\bar{H} = \frac{1}{n} \cdot \sum_{i=1}^n H_i$ $s_H = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (H_i - \bar{H})^2}$	H_i acc. EN ISO 6506, part 2, No. 5.6	Single measurements: 246.0–245.0–246.0–246.0–246.0 $\bar{H} = 245.8 / s_H = 0.45$

STEP	Sources of uncertainty	Abbreviation	Formula	Literature/Certificate	Sample calculation [..] = HBW 2.5/187.5
6	Standard uncertainty of hardness testing machine when measuring CRM	$u_{\bar{H}}$	$u_{\bar{H}} = \frac{t \cdot s_H}{\sqrt{n}}$	t=1.15 (Student-factor) for n=5 and $\square=68.3\%$	$u_{\bar{H}} = \frac{1.15 \cdot 0,45}{\sqrt{5}} = 0.23$
7	Mean value and standard deviation of the testing of a sample	\bar{x} , s_x	$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^n x_i$ $s_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$	x_i : 5 measurements on sample	Single measurements: 288.0–290.0–285.0–285.0–282.0 $\bar{x} = 286.0$ $s_x = 3.08$
8	Standard uncertainty when measuring a sample (including sample preparation, e.g. polishing)	u_x	$u_x = \frac{t \cdot s_x}{\sqrt{n}}$	t=1.15 (Student-factor) for n=5 and $\square=68.3\%$	$u_x = \frac{1.15 \cdot 3.08}{\sqrt{5}} = 1.59$
9	Calculation of the expanded measurement uncertainty	U	$U = 2 \cdot \sqrt{u_E^2 + u_{X_{CRM}}^2 + u_{CRM}^2 + u_H^2 + u_x^2}$	steps 1 to 8, 95% confidence level	$U = 2 \cdot \sqrt{1.76^2 + 0.75^2 + 0.39^2 + 0.23^2 + 1.59^2}$ $U = 5.1$
10	Overall result		$\bar{X} = \bar{x} \pm U$	steps 7 and 9 95%	$\bar{X} = 286.0 \pm 5.1$
ALTERNATIVE (in %)					
11 A	Calculation of the relative expanded measurement uncertainty	\tilde{U}	$\tilde{U} = \frac{U}{\bar{X}_{CRM}} \cdot 100\%$	steps 1 to 9, 95%	$\tilde{U} = \frac{5.1}{246.8} \cdot 100\% = 2.1\%$
12 A	Overall result		$\bar{X} = \bar{x} \pm U$	steps 7 and 11A 95%	$\bar{X} = 286.0 \text{ HBW } 2.5/187.5 \pm 2.1\%$

Example 3

Determination of emission measurement and sampling uncertainties estimated by well experienced experts

In 1998 an interlaboratory estimation was carried out among 26 institutes from Germany and Switzerland measuring air polluting emissions according to VDI technical method standards. The results are given in [83] "Verlauf und Ergebnisse einer geordneten Ringschätzung der Unsicherheiten von Emissionsmessungen" (Process and results of an well-ordered interlaboratory estimation of the uncertainties from emission measurements) by Janusz S. Morkowski, Umweltbundesamt Texte 54/99, Berlin, 1999.

Five popular and well known VDI technical methods of emission measurement had been chosen. Each of them consist of several procedural steps that may influence the final result of the measurement. The methods chosen were:

- 1) VDI 2066, Blatt 2: Filterhead with stuffing tube for dust determination (gravimetrically)
- 2) VDI 2066 Blatt 7: Filterhead with a plane filter for dust determination
- 3) VDI 2456 Blatt 10: Total nitrogenoxides
- 4) VDI 3480 Blatt 1: HCl with different analytical techniques
- 5) VDI 3481 Blatt 2: Total organic carbon by sorption method.

The laboratories received a questionnaire specifying the procedure of the estimation and were instructed by the organiser.

Uncertainty components for the different steps of the procedures (methods) were to be estimated according to an estimation form. Those steps included planning, preparation, sampling, determination of the gas volume, sample transportation, preparation of the sample, measurement and analysis in the laboratory, evaluation of the result.

These single uncertainty components of the procedure were combined according to the uncertainty propagation law of Gauss to the total measurement uncertainty U_T .

Estimations were made both for the "standard case", S-case, which can be expected for about two thirds of the tests and for the "problematical case" X-case, which should also cover most problematic cases (e.g. evil conditions, sampling not representative, ...) with a probability of 95% and more.

75 sampling personnel and 20 employees of the laboratories participated in the estimation test being completely independent from each other. The following table gives the results of the estimation test in % (for the S-case). For methods 3 and 4 only the results for the photometrical analysis are given here. The entire tables of the results can be found in [83].

As it can be seen from the table the estimated total uncertainties for the 5 different methods vary from 13% to 21% with the sampling step being one of the major components.

The total uncertainties are calculated first for all steps of the procedure $U_T(I)$ and secondly also for the case where the planning step is not included in the calculation $U_T(II)$. In the latter case the uncertainties are reduced especially for the filter tests. The relations of X-case to S-case are approximately 2.7.

For one of the methods, the dust determination with plane filters, also practical interlaboratory comparison data were available. Laboratory intercomparisons at the Emission-Simulation-Plant at the HLfU at Kassel from 1994 to 1996 give a mean standard deviation of 15.4% for small concentrations of dust in emissions.

Because planning, which is usual in regular emission measurements is not needed in this case, the estimated uncertainty data $U_T(II)$ of the dust determination with plane filters without the planning step can be used for comparison. The value estimated in the interlaboratory estimation is 15.5%, which is nearly equal to the value obtained in the HLfU-Plant.

Method standard	VDI 2066 Bl.2	VDI 2066 Bl.7	VDI 2456 Bl. 10	VDI3480 Bl. 1	VDI 3481 Bl.2
Measurand	Dust	Dust	NO+NO ₂	HCl	TOC
Analytical technique:	gravimetric	gravimetric	photometric	photometric	sorption
Planning	11.06	8.27	3.59	3.32	3.26
Preparation of sampling	6.44		6.85	3.56	5.43
Sampling	12.55	11.46	10.11	6.21	15.97
Transport and sample preparation	2.34	2.61	8.15	3.31	5.38
Preparation of sample / treatment of the filter /sorption material	3.61	8.09			10.06
Determination of the volume	4.33	4.21		4.33	3.87
Additional parameters for volume determination	3.68	4.32		2.83	3.73
Other influences	0.46	0.67		0.41	0.75
Analytical measurement			8.37	8.11	
Other influences			1.12		
Total uncertainty U_T(I) From these mean values of the single uncertainty components	19.3	17.6	17.3	12.9	21.3
Total uncertainty U_T(II) as above without the planning step	15.8	15.5	16.9	12.5	21.1
Relation X-case/S-case	3.04	2.93	2.39	2.64	2.62

Though the single estimation results (which are not given here) are distributed widely, the results of the estimation (average and the limits of a rather narrow range of confidentiality) may be a useful approach for the determination of measurement uncertainties in this field. The results also show that it is important that the estimator knows the testing procedure well, that he is very experienced in the field concerned and that the details on the conditions of the measurement as a basis for the estimation are provided.

The estimation of the uncertainties for the sampling step of the different methods provide examples which may serve as a basis also for other methods.

As it can be seen from the table the estimated total uncertainties for the 5 different methods vary from 13% to 21% with the sampling step being one of the major components.

The total uncertainties are calculated first for all steps U_T(I) and secondly also for the case where the planning step is not included U_T(II). In the latter case the uncertainties are reduced especially for the filter tests. The relations of X-case to S-case are approximately 2.7.

For one of the methods, the dust determination with plane filters, also practical interlaboratory comparison data were available. Laboratory intercomparisons at the Emission-Simulation-Plant at the HlfU at Kassel from 1994 to 1996 give a mean standard deviation of 15.4% for small concentrations of dust in emissions.

Because planning is not needed in this case, the estimated uncertainty data U_T(II) of the dust determination with plane filters without the planning step can be used for comparison. The value estimated in the interlaboratory estimation is 15.5%, which is well comparable to the practically obtained value.

Though the single estimation results (which are not given here) vary widely, the results show that estimation may be a useful approach for determination of measurement uncertainties. The results also show that it is important that the estimator knows the testing procedure well and is provided details on the conditions as a basis for the estimation.

The estimation of the uncertainties for the sampling step of the different methods provide examples which may serve as a basis for other methods.

Example 4

Determining the measurement uncertainty in preparing a calibration standard by the mathematical analytical approach

Useful examples for measurement uncertainty from the single components, closer following the strict mathematical analytical way, can be found in references [2] and [3].

The following example leans on Example A1 of the EURACHEM /CITAC Guide "Quantifying Uncertainty in Analytical Measurement" [3]. It has been chosen to clarify the 8 steps of the procedure.

The aim is to prepare a calibration standard of Cd in HNO₃ for AAS. To obtain the Cd calibration standard of ca. 1000mg/l the following procedure was applied:

1. Weighing a piece of metal (surface cleaned).
2. Dissolving the metal in a 100 ml flask by adding 1 ml HNO₃ and filling with deionised water.

Step 1) Specify measurand, express mathematically the equation relating measurand and input quantities. Identify all uncertainty sources.

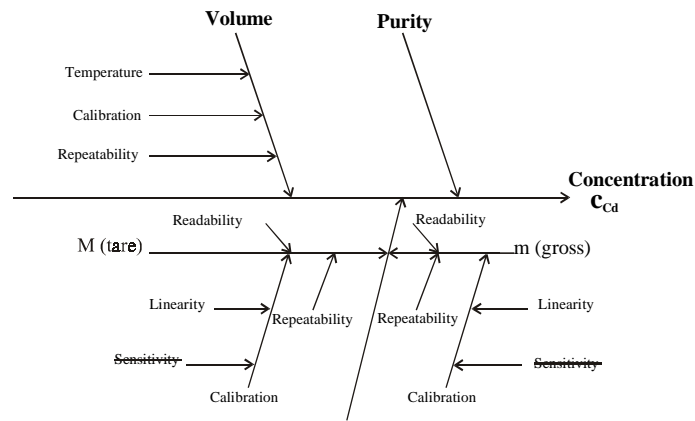
$$c_{Cd} = \frac{1000 \cdot m \cdot P}{V} [mg/l]$$

- c_{Cd} : concentration of the calibration standard obtained
 m : mass of the clean high purity Cd piece [mg]
 P : Purity of the metal
 V : volume of the flask [ml]
1000: conversion factor from ml to l

Listing the components of uncertainty

1. Purity of the Cd: supplier's certificate: 99.99 ± 0.01 %.
2. Mass of the metal from weighing in the flask. The piece weighed 0.10028g. The manufacturer's literature identifies 3 uncertainty sources for tare weighing:
 - 2.1 repeatability,
 - 2.2 readability of the balance scale,
 - 2.3 calibration (involving sensitivity of the balance and linearity).Sensitivity can be neglected because weighing was done on the same balance over a narrow range. Buoyancy correction can be neglected [3] being very small.
3. Volume of the solution: 100ml. Uncertainty sources are:
 - 3.1 uncertainty in the certified internal volume of the flask
 - 3.2 Filling the flask to the mark
 - 3.3 Temperature influences

"Cause and effect" diagram:



Step 2) Determine the input quantities

1. Purity of the Cd: 99.99 ± 0.01 % i.e. 0.9999 ± 0.0001 .
2. Mass of the Cd: 0.10028 g.
3. Volume of the solution: 100ml

Step 3) Quantifying the single uncertainty components

1. Purity of the Cd: Type B evaluation: 99.99 ± 0.01 % i.e. 0.9999 ± 0.0001 .
A rectangular distribution is assumed, because there is no further information. Therefore the standard uncertainty of the purity is:

$$u(P) = \frac{0.0001}{\sqrt{3}} = 0.000058$$

2. Mass of the Cd: 0.10028 g. The manufacturer of the balance recommends 0.05 mg as uncertainty estimation, this value can be taken directly as

$$u(m) = 0.05mg \text{ (Type B)}$$

3. Volume of the solution: 100ml

3.1 uncertainty in the certified internal volume of the flask: the manufacturer quotes a volume for the flask of 100 ± 0.1 ml at 20°C .

No confidence level is given, so a triangular distribution was chosen, because in an effective production process the nominal value is more likely than extremes. Therefore

$$u(V_1) = \frac{0.1ml}{\sqrt{6}} = 0.04ml \text{ (Type B)}$$

3.2 Filling the flask to the mark: An experiment of 10 fill and weigh experiments gave a standard deviation of 0.02 ml. This can be used directly as

$$u(V_2) = 0.02ml \text{ (Type A)}$$

3.3 Temperature influences: The laboratory temperature varies between the limits of $20^\circ\text{C} \pm 4$. The volume expansion of water is large compared to flask material, which is therefore neglected. The volume expansion of water is $2.1 \cdot 10^{-4} / ^\circ\text{C}$, leading to volume variation of

$$\pm (100 \cdot 4 \cdot 2.1 \cdot 10^{-4})ml = \pm 0.084ml$$

Assuming rectangular distribution gives

$$u(V_3) = \frac{0.084}{\sqrt{3}} = 0.05ml$$

The three volume effects add to each other and are treated like a sum. The combined uncertainty from volume effects is then

$$u(V_{total}) = \sqrt{0.04^2 + 0.02^2 + 0.05^2} = 0.07 \text{ ml}$$

Step 4) Identify the covariances (of correlated input quantities)

Correlation effects are not known and the approximation is made that there is no correlation.

Step 5) Calculate the result of the measurement from the input quantities

Determination of the concentration

$$c_{Cd} = \frac{1000 \cdot m \cdot P}{V} [\text{mg} / \text{l}] = \frac{1000 \cdot 100.28 \cdot 0.9999}{100} \text{ mg} / \text{l} = 1002.7 \text{ mg} / \text{l}$$

The concentration of the calibration standard is 1002.7 mg/l.

Step 6) Calculate the combined uncertainty

Because the above equation is a multiplicative expression, the uncertainties are combined by:

$$\frac{u_{combined}(c_{Cd})}{c_{Cd}} = \sqrt{\frac{u(P)^2}{P^2} + \frac{u(m)^2}{m^2} + \frac{u(V_{total})^2}{V_{total}^2}} = \sqrt{\frac{0.000058^2}{0.9999^2} + \frac{0.05^2}{100.28^2} + \frac{0.07^2}{100^2}} = 0.0009$$

$$u_{combined}(c_{Cd}) = 0.9 \text{ mg} / \text{l}$$

Comparing the uncertainties from the components shows that volume and mass uncertainties contribute in a similar way to the overall uncertainty, while the purity has almost no influence on it.

Step 7) Calculate the expanded uncertainty

The expanded uncertainty is

$$U = k \cdot u_{combined}(c_{Cd}) = 2 \cdot 0.9 \text{ mg} / \text{l} = 1.8 \text{ mg} / \text{l}$$

The coverage factor k is chosen to be 2 as recommended by the GUM [1].

Step 8) Give the result together with the uncertainty as estimated

The concentration of the Cd standard is 1002.7 ± 1.8 mg/l. The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor k = 2, providing a level of confidence of approximately 95%.

5. Conclusions

There are many possibilities and ways for uncertainty estimation of methods or components thereof, based on experience and general knowledge gathered from practice.

Estimates can also be based on a list of the main influencing factors. Such a list is helpful in any case, as it facilitates final checking of the estimation method for covering all relevant components.

The laboratory's expertise is needed to decide whether the respective data can be used for the uncertainty estimation with view to: observed range, whether all major components are included, e.g. preparation steps, dilution, It may be difficult to put all these components and the available data together into a model. The assessment of measurement uncertainties can only be based on the present state of the art, however. Laboratories should use the data available for the evaluation of measurement uncertainty.

The degree of rigor and detail of mathematical models differs widely among fields of measurement. In particular in the testing area, it is often not possible to draw up a very detailed model, as the method does not allow it, or the method does not require it. In those cases, a much simpler approach may be applied (ISO/IEC 17025, 5.4.6.2). Furthermore, the evaluation of measurement uncertainty is still evolving, and as a result there are great differences in the progress made in the various fields of measurement and testing. Therefore in different fields uncertainty estimations will be at a different stage of development, and the realisation may need different amounts of time and may need to be implemented stepwise.

It is often argued, that customers are confused by uncertainty statements. Therefore ISO/IEC 17025 (5.10.3) states, that measurement uncertainty does not always have to be stated but only in those cases where it is relevant for the client.

Finally, it should be realised that measurement uncertainty is of added value, both for the laboratory in terms of its quality management and for the customer as well.

Still it is not the task of laboratories alone to fulfil uncertainty requirements:

- Accreditation bodies are asked to explain what they expect from laboratories. Education of the assessors is very important. They will have to look into the uncertainty statements and check whether laboratories can comply with them. This will be especially important with view to a fair competition between laboratories.
- Manufacturers of instruments and reference material are required to deliver useful data, in particular concerning realistic and "GUM-compliant" uncertainty statements.
- Normalisation bodies (e.g. CEN, ISO) should give detailed information on uncertainties from method validation with new standards.
- Clients have to get information on measurement uncertainty. The leaflet "Important information to our customers concerning the quality of measurements" [18] published by SP (Swedish National Testing and Research Institute) and other organisations may be a helpful tool for clients to understand the meaning of measurement uncertainty.

6. Compilation of the main and some specific literature on the field of measurement uncertainty

	Title	Author	Year	Remarks	Source
Basic literature including methods for practical determination of measurement uncertainties					
[1]	Guide to the expression of the uncertainty in measurement	BIPM/IEC/IFCC/ISO/OIML/IUPAC/IUPAC ISBN 92 67 10188 9	1993 / 1995	"GUM", main document on uncertainty, establishes general guidelines for evaluating and expressing measurement uncertainty	ISO 110 p., approximately CHF 92,00
[2]	Expression of the Uncertainty of Measurement in Calibration	EA-4/02, EA Taskforce for Revision of WECC doc 19-1990	1999	Very good, comprehensible document. Many well explained examples from calibration field.	Can be downloaded from EA homepage (www.european-accreditation.org)
[3]	Quantifying Uncertainty in Analytical Measurement, EURACHEM /CITAC Guide	Eurachem / CITAC	1995 / 2000	Most comprehensible document for measurement uncertainties. Many very good examples. Good explanation of "Spreadsheet method".	120 p.; download from www.measurementuncertainty.org or www.eurachem.bam.de
[4]	The Expression of Uncertainty and Confidence in Measurement	NAMAS NIS 3003	1997	Good description of the concepts of uncertainty determination, examples mainly from calibration area.	NAMAS, United Kingdom Accreditation Service, 21-47 High Street, Feltham, Middlesex TW13 4UN
[5]	Guidelines for evaluating and expressing uncertainty of NIST measurement results	Barry N. Taylor and Chris E. Kuyatt, NIST	1993	Good description of the concepts of uncertainty determination	Download from http://physics.nist.gov
More information on uncertainty determination					
[6]	Estimating Uncertainties in Testing Measurement Good Practice Guide Nr. 36	Keith Birch, BMTA	2001	Very good, with examples	BMTA, Teddington, Middlesex, UK, TW11 0NQ, ISSN 1368-6550
[7]	The NIST Reference on Constants, Units, and Uncertainty	NIST homepage	2000	Very good, short introduction on to how to evaluate and calculate uncertainties.	http://physics.nist.gov/cuu/Uncertainty/index.html
[8]	Estimating Uncertainties in Testing, A Guide to Estimating and reporting Uncertainties of Measurement in Testing	BMTA	1995		BMTA, PO Box 101, Teddington, Middlesex, TW11 0NQ, United Kingdom Tel: +44 (0)20 8943 5524
[9]	NPL – Measurement Good Practice Guide, National Physics Laboratory, A beginners guide to Uncertainty in Measurement	NPL , Stephanie Bell	1999	Good, simple introduction for beginners. Some chapters free on internet.	NPL 25 £, free chapters: http://www.npl.co.uk/npl/publications/good_practice/uncert/
[10]	Determining and Reporting Measurement Uncertainties	National Conference of Standards Laboratories RP 12	1995		30\$, USA, Boulder CO; http://www.ncslinternational.org/publications/pubs-list.cfm
[11]	U.S. Guide to the Expression of Uncertainty in Measurement	ANSI/NCSL Z540-2-1997	1997		50\$, USA, Boulder CO; http://www.ncslinternational.org/publications/pubs-list.cfm
[12]	ISO/BIPM-Leitfaden:Meßunsicherheit	Dr. W. Kessel	1998	Example of a weighing experiment, in German.	www.metrodata.de/papers/waage.html
[13]	Estimating measurement uncertainty: reconciliation using a cause and effect approach	S.L.R. Ellison, V.J. Barwick; Accred. Qual. Assur. 3, P.101-105	1998		

[14]	Swiss Accreditation Service: Dok. 706.d "Validierung von Prüfverfahren"	Swiss Accreditation Service: Dok. 706.d	1995		
[15]	French Standardisation: Aid in the procedure for estimating and using uncertainty in measurement and test results – FD X 07-021	AFNOR		Description how to use precision data for uncertainty estimation	
Implementation of uncertainty					
[16]	Uncertainty of test results ("Result Uncertainty")	DAR-EM22, DAR ATF Ad-hoc group "Uncertainty in testing"	1996	General notes on application of uncertainty. Good table of publications on uncertainty.	DAR-EM22, http://www.dar.bam.de/ under documents, ATF
[17]	The Expression of Uncertainty in Quantitative Testing	EA-3/02, (previously EAL-G23)	1996	Guidance for accreditors for implementation of uncertainty requirements	http://www.european-accreditation.org/
[18]	Important information to our customers concerning the quality of measurements	SP, P.O. Box 857, SE-501, 15 Boras, Sweden, Tel: +46-33-13-55-02	2000	Leaflet on measurement uncertainty in practice	SP, P.O. Box 857, SE-501, 15 Boras, Sweden, Tel: +46-33-13-55-02 info@sp.se
[19]	Measurement Uncertainty	UKAS	2000	Short introduction to measurement uncertainty	www.ukas.com Accreditation topics: Measurement Uncertainty
[20]	ILAC-G17:2002: Introducing the Concept of Uncertainty of Measurement in Testing in Association with the Application of the Standard ISO/IEC 17025	ILAC and EA	2001	Important policy	www.ilac.org
[21]	APLAC Policy and Guidance on the estimation of Uncertainty of Measurement in Testing – Draft November 2001	APLAC	2001	Good and useful document, gives sector oriented advice	
[22]	A2LA policy	A2LA	2000	An implementation strategy into accreditation	www.a2la.org
[23]	Assessment of Uncertainties of Measurement for Electrical Testing	Nata Australia	1992		
Standards associated with measurement uncertainty					
[24]	ISO/TS 14253-2:1999 Geometrical Product Specifications (GPS) - Inspection by measurement of workpieces and measuring equipment -- Part 2: Guide to the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification	ISO	1999		ISO, www.iso.ch ; CHF 164.00
[25]	ISO 3951:1989 Sampling procedures and charts for inspection by variables for percent nonconforming	ISO	1989		ISO, www.iso.ch ; CHF 188.00
[26]	ISO 6974 Natural gas -- Determination of composition with defined uncertainty by gas chromatography	ISO	2000		
[27]	ISO 13752 Air quality -- Assessment of uncertainty of a measurement method under field conditions using a second method as reference	ISO	1998		
[28]	ISO 7066 Assessment of uncertainty in calibration and use of flow measurement devices	ISO	1997		
[29]	ISO 5725-1-6: 1994/Cor. 2001 Accuracy (trueness and precision) of measurement methods and results	ISO	1994		ISO, www.iso.ch
[30]	DIN 40080 Stichprobenprüfung anhand qualitativer Merkmale DIN ISO 2859-1, Ausgabe:1993-04 Annahmestichprobenprüfung anhand der Anzahl fehlerhafter Einheiten oder Fehler (Attributprüfung)	DIN	1993		Beuth Verlag
[31]	DIN 53804 T3/T4 Statistische Auswertungen, Ordinal-/Attributmerkmale	DIN	1982 / 1985		Beuth Verlag
[32]	DIN 25424 Fehlerbaumanalyse	DIN	1990		Beuth Verlag

[33]	DIN 1319 Teil 3 "Auswertung v. Messungen einer einzelnen Meßgröße"; Meßunsicherheit; Teil 4 "Behandlung von Unsicherheiten bei der Auswertung von Messungen"	DIN	1996 / 1999		Beuth Verlag
[34]	Statistical assessment of the uncertainty of measurement results: Guide to the use of repeatability, reproducibility and trueness estimates in measurement uncertainty estimation	ISO TC69/SC6/WG7	2001		Draft
Software for Uncertainty calculation					
[35]	Tools for the test laboratory to implement measurement uncertainty budgets	Sven Nytoft Rasmussen; Nordtest Techn. Report 430	1999	Very good comparison of the concepts of the different computer programs	Nordtest, P.O. Box 116, FIN-02151 Espoo, Finland, Phone +358-0-455-4600
[36]	MUSAC (Measurement Uncertainty in Analytical Chemistry)	EMPA, Creasoft AG and others	2001	software system for application in the chemical laboratory: e.g. titration, HPLC, GC, GC/MS, ICP/OES, ICP/MS and AAS, first part available September 2001	information at: www.musac.ch ; price not jet known
[37]	AESoft Uncertainty	Atkinson Engineering, Inc			1000 \$, free demo version; www.aesoft.com/Unc1.html
[38]	GUM Workbench	Metrodata GmbH, Grenzach-Wyhlen, Germany			1100 \$, free demo version; Metrodata GmbH; http://www.gum.dk ; http://www.metrodata.de/
[39]	DFM-GUM	Danish Institute of fundamental metrology			300 \$; http://www.dfm.dtu.dk/en/consult/dfm-gum.htm
[40]	Uncertainty Analyzer	Integrated Sciences Group. Uncertainty Analyser 1.0 manual	1994-96		700 \$, USA, Bakersfield, Ca. www.isgmax.com ; http://www.quametec.com/downloads.htm
[41]	Expression Buddy	James E. Presley and Daniel B. Presley			free software; download at http://www.jpresley.com/
Books					
[42]	Experimentation and Uncertainty Analysis for Engineers	Hugh W.Coleman, W. Glenn Steel	1999		New York, Wiley & Sons Inc. ISBN 0-471-12146-0
[43]	Meßunsicherheit und Meßdatenauswertung	Klaus Weise, Wolfgang Wöger	1999		Weinheim, Wiley-VCH; ISBN: 3-527-29610-7
[44]	Messunsicherheiten: Theorie und Praxis	Franz Adunka	1998		Essen, Vulkan-Verlag, ISBN 3-8027-2186-1
[45]	Estimer l'incertitude, Mesures - Essais	Christophe Perruchet, Marc Priel	2000		Adam Hilger, Bristol; ISBN: 0-7503-0060-4
[46]	Uncertainty, calibration, and probability; the statistics of scientific and industrial measurement	C. F. Dietrich	1991		North-Holland, Amsterdam, ISBN: 0-444-87796-7
[47]	Uncertainty models for knowledge-based systems : a unified approach to the measurement of uncertainty	Irwin R. Goodman and Hung T. Nguyen	1985		Instr. Soc. Of America, ISBN: 1-55617-628-7
[48]	Measurement uncertainty: methods and applications	Ronald H. Dieck	1992, 1997		Wiley-VCM ISBN: 3-527-28748-5
[49]	Handbuch Validierung in der Analytik	Stavros Kromidas	1999		
[50]	Calculation & Reporting Uncertainties of Measurements in Testing	N. Kukadia, Tenby Industries Limited, Birmingham, UK	1996		

Some examples of special topics

[51]	Guidelines for expressing the Uncertainty of Measurement results containing uncorrected Bias	Philips, Eberhardt and Parry, NIST; Journal of Research of the Nat. Inst. Of Standards and Technology, 102, S. 577	1997	Taking into account uncorrected bias	download from http://nvl.nist.gov/pub/nistpubs/jres/102/5/j25phi.pdf
[52]	Calculation of Measurement Uncertainty Using Prior Information	S.D. Phillips, W.T. Estler, M.S. Levenson, K.R. Eberhardt	1998	Treating bias in uncertainty evaluation	download from http://nvl.nist.gov/pub/nistpubs
[53]	Uncertainty and Dimensional Calibrations	Doiron and Stoup, Journal of Research of the Nat. Inst. Of Standards and Technology, 102, S. 647	1997	Examples for variables contributing in calibration	download from http://nvl.nist.gov/pub/nistpubs
[54]	An Interpretation of the Guide to the expression of Uncertainty in Measurement	R.N. Kacker, NIST Techni-Pubs SP 500-244	2000		20\$, ordering at http://nvl.nist.gov/pub/nistpubs
[55]	Observing validation, uncertainty determination and traceability in developing Nordtest test methods	Magnus Holmgren, NT Techn. Report 403	1998		www.vtt.fi/nordtest/tec403.htm
[56]	Uncertainty of pH Measurements	H. Jensen, L. Nielsen, Nordtest Techn Report 284	1995		Nordtest, P.O. Box 116, FIN-02151 Espoo, Finland, Phone +358-0-455-4600
[57]	Traceable calibration and uncertainty of measurements in scanning electron microscopy	Halldur Gudmundsson, Thomas Runarsson, Nordtest Techn Report 252	1994		Nordtest, P.O. Box 116, FIN-02151 Espoo, Finland, Phone +358-0-455-4600
[58]	Traceable calibration and uncertainty of measurements and tests	B. Steffen, H.Kallio, Freygardur Thorsteinsson, Nordtest Techn. Report 251	1994		Nordtest, P.O. Box 116, FIN-02151 Espoo, Finland, Phone +358-0-455-4600
[59]	Calibration, traceability and uncertainty	Nordtest-Seminar in Espoo, NT Techn. Report 305	1995		
[60]	A view on the assessment of the technical competence of testing laboratories	Jarl Forstén, Nordtest Techn Report 149	1991		
[61]	The use of uncertainty estimates in testing	Nordtest SP Report 1993:47	1993		
[62]	Nordtest's Views on Measurement in Metrology and Testing	Nordtest, Position Paper	1994		Nordtest, P.O. Box 116, FIN-02151 Espoo, Finland, Phone +358-0-455-4600
[63]	Uncertainty - to a certain level	Nordtest Position paper 005	1998	Ways for assessing uncertainties for different situations of tests and measurement	Nordtest, P.O. Box 116, FIN-02151 Espoo, Finland, Phone +358-0-455-4600
[64]	Estimating Measurement Uncertainty	Erwin Achermann, Oscar Chinellato; Technical Report 346. ETH Zürich	2000	Theoretical Scientific Computing: The "Maximum Likelihood (fitting of a) Functional Relationship" (MLFR)	ETH Zürich, Institute of Scientific Computing; http://www.inf.ethz.ch/research/wr/publications/tr.html
[65]	Instrument and Apparatus - Measurement Uncertainty, part 1	ANSI/ASME PTC 19.1-1985 (1990)	1985 / 1990		ASME, New York
[66]	Measurement uncertainty of radiated emissions	David A. Hill, Motohisa Kanda	1997		
[67]	The fitness for purpose of analytical methods	Eurachem	1998	Method validation studies	ISBN 0-948926-12-0
[68]	ILAC-G13. Guidelines for the requirements for the Competence of Providers of Proficiency Testing Schemes (uncertainty assessment in the Annex)	ILAC	2000		http://www.ilac.org/
[69]	ILAC-G8. Guidelines on Assessment and Reporting of Compliance with Specifications	ILAC		Rules for supplier and client concerning the compliance of a product	http://www.ilac.org/
[70]	NAMAS NIS 80	NAMAS NIS 80			

[71]	Uncertainties of measurement for NAMSA Electrical product testing laboratories NIS 20, NAMAS	NAMAS NIS 20	1989		
[72]	Estimation and Expression of Measurement Uncertainty in Chemical Analysis	Nordic Committee on Food Analysis, NMKL Procedure 5	1997		
[73]	Measurement uncertainty considerations for co-ordinate measuring machines	S.D. Phillips, B. Borchardt, G. Caskey	1993		
[74]	Step by step estimate of uncertainty of test results	J.S. Morowski, EMPA, Switzerland, at 2nd EUROLAB Symposium "Testing for the year 2000", Florence, April 1994	1994	Examples of estimation of uncertainties from practice	EUROLAB
[75]	Assessment and Practical Use of Uncertainties in test results	Hans Andersso, SP, Sweden at 2nd EUROLAB Symposium "Testing for the year 2000", Florence, April 1994	1994	Calculating measurement uncertainties	EUROLAB
[76]	Uncertainty in testing	Norbert Müller, arsenal research, Austria	1999	Important aspects of harmonisation of the use of uncertainties	EUROLAB - in discussion
[77]	Practical Experiences with Uncertainty Evaluation at VTT Chemical Technology	Veikko Komppa, VTT, Finland at 2nd Workshop "Measurement Uncertainty in Chemical Analysis", Berlin September 1997	1997	Examples from practice, graphics: measurement uncertainty against concentration	Eurachem/D / EUROLAB-D
[78]	Interlaboratory Study and Validation Data	Stephen L.R. Ellison, Laboratory of the government Chemist, UK at 2nd Workshop "Measurement Uncertainty in Chemical Analysis", September 1997	1997	Use of interlaboratory comparisons for uncertainty determination	Eurachem/D / EUROLAB-D
[79]	Autosamplers - a major uncertainty factor in HPLC Analysis Precision	S. Küppers, B. Renger, V.R. Meyer, LC,GC Europe, 2/2000	2000		
[80]	The evaluation of the uncertainty in knowing a directly measured quantity	I.H. Lira, W. Wöger, PTB, Braunschweig, Germany; Pontificia Universidad Catolica de Chile, Santiago	1998		
[81]	Is the estimation of measurement uncertainty a viable alternative to validation?	Meas. Sci. Technol. 9, 1167-1173	1998		
[82]	NIST list of recent and new documents	S. Küppers, Accred. Qual. Assur. 3 1998 p. 412-415 NIST homepage, search option		Good variety of publications, e.g. special literature on measurement uncertainty, some can be downloaded See Example 3 of this document	http://nvl.nist.gov/pub/nistpubs/nistpubs.htm 7,67 Euro; www.umweltbundesamt.de
[83]	Verlauf und Ergebnisse einer geordneten Ringschätzung der Unsicherheiten von Emissionsmessungen (Process and results of an ordered interlaboratory estimation of the uncertainties from emission measurements)	J.S. Morkowski, Umweltbundesamt, Texte 54/99, Berlin, 1999			